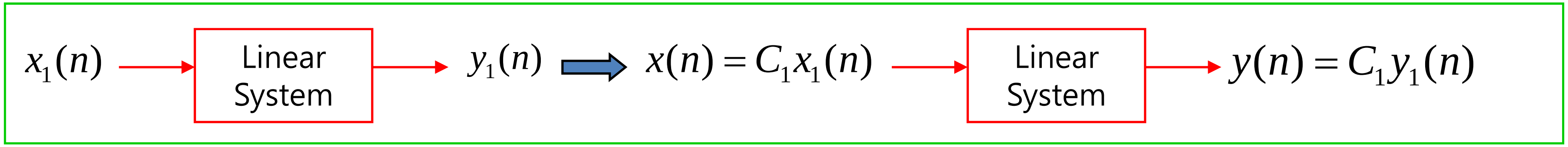
# 선 형 변 환

## Discrete System

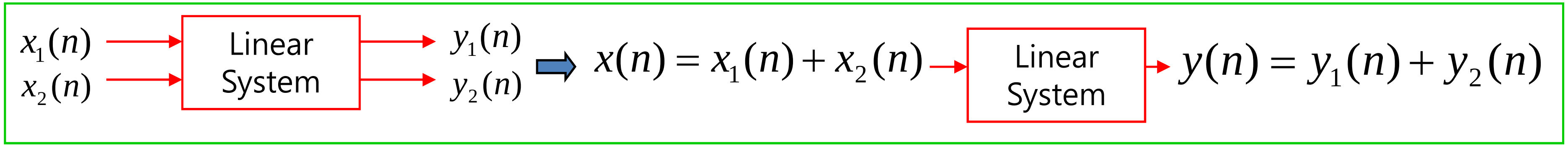
## - Linearity

A system is Linear ⟷ (if and only if) it satisfies the principle of homogeneity and additivity ⇒ Superposition

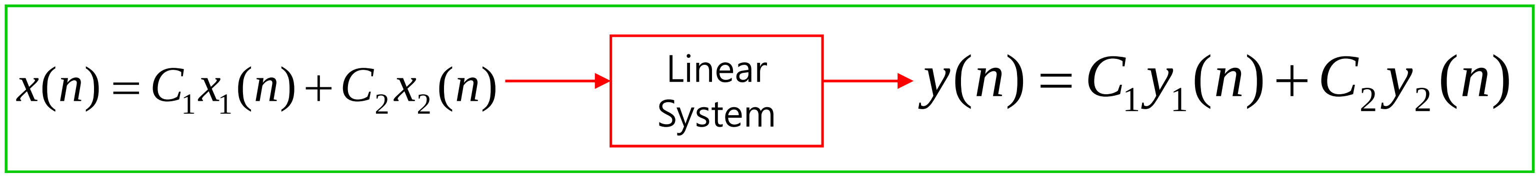
### Homogeneity



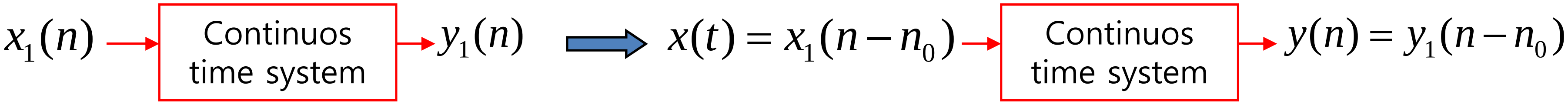

### Additivity




### Superposition



## - Time Invariance




## - Linear Time Invariance (LTI)

When a system if both linear and time invariant,
it is called a linear time invariant (LTI) system

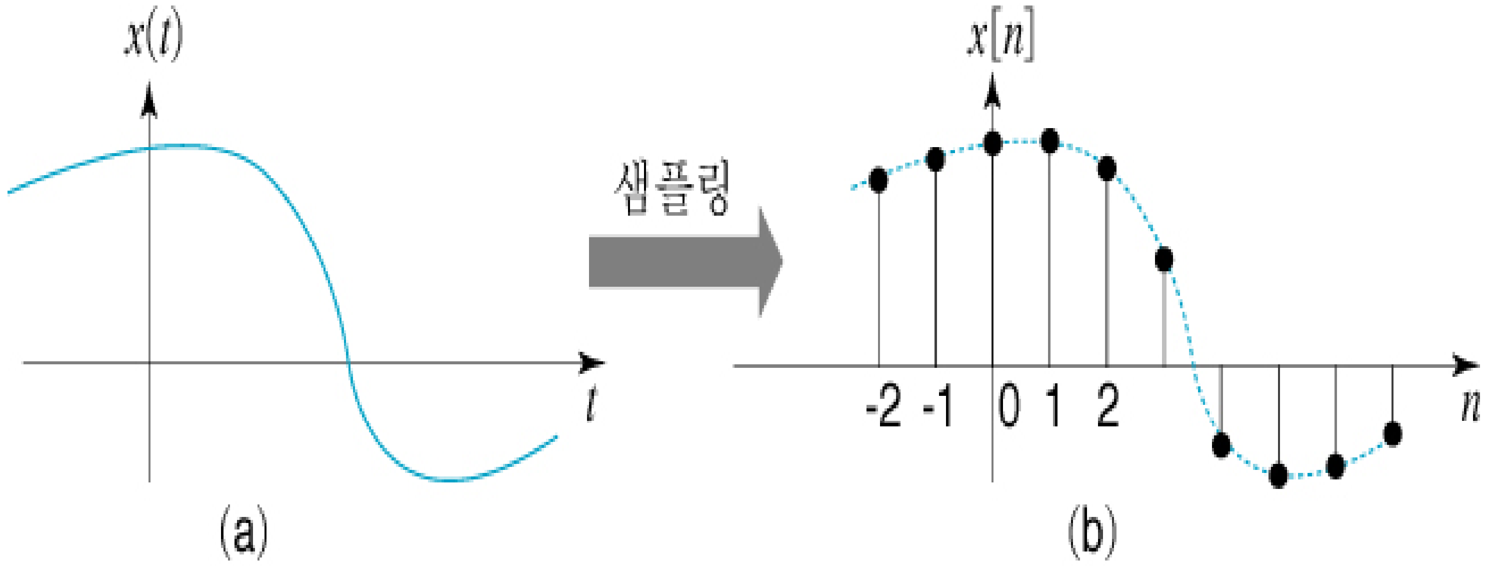
## - Causality

its current output depends on past and current inputs
but not on future input.

## - Stability

if the system input is bounded,
and if the system output is also bounded, it is called that
the system is stable (BIBO)

## - Discretize



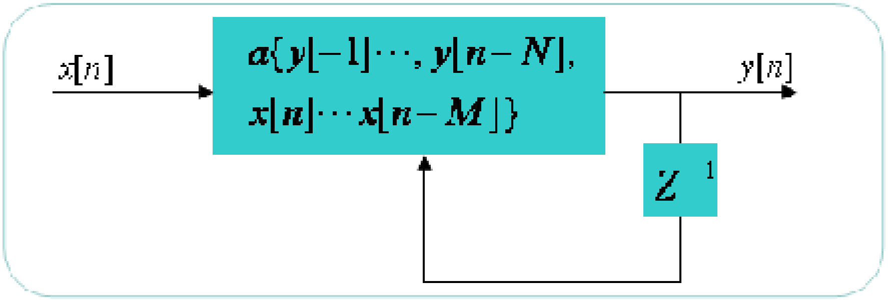

연속신호

이산신호

## - Recursive AND Nonrecursive

Recursive (재귀형)

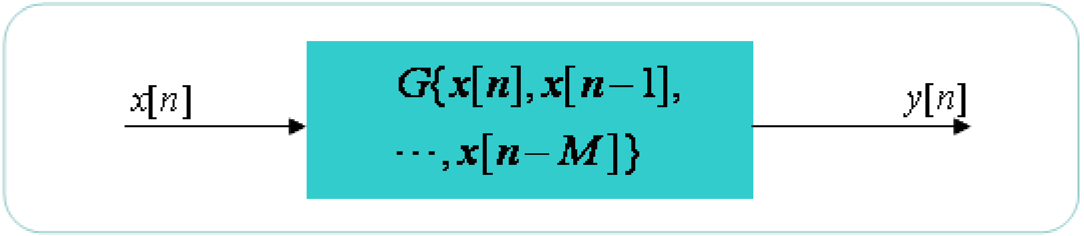
$$y(n)+\sum_{k=1}^{N}a_k y(n-k)=\sum_{k=0}^{L}b_k x(n-k)$$




The Output depends on previous values of the output as well as on the input.

Non-Recursive (비재귀형)

$$y(n)=\sum_{k=0}^{L}b_k x(n-k)$$




The Previous values of the output do not enter the calculations.

## - 선형 차분방정식의 일반적인 해법 1)

$$y[n] = -\sum_{k=1}^{N} a_k y[n-k] + \sum_{k=0}^{M} b_k x[n-k]$$

$$y[0] = -\sum_{k=1}^{N} a_k y[-k] + \sum_{k=0}^{M} b_k x[-k]$$

$$y[1] = -\sum_{k=1}^{N} a_k y[1-k] + \sum_{k=0}^{M} b_k x[1-k]$$

만약, 초기값을 알 수 있다면, n=1,2,3,4,5, 이렇게 계속 대입하며 그 결과를 찾아볼 수 있다.
그러나 이 방법은 출력(y)을 수학적 표현으로 나타내는데 한계가 있다.

## - 선형 차분방정식의 일반적인 해법 2)

$$y[n] = -\sum_{k=1}^{N} a_k y[n-k] + \sum_{k=0}^{M} b_k x[n-k]$$

$$y[n] = y_h[n] + y_p[n]$$

균일해

특이해 → (선형 미방처럼) 입력의 형태에 따라 변동

(입력신호 $x[n]=0$이라고 가정하면,)

$$\sum_{k=0}^{N} a_k y[n-k] = 0$$

($y_h[n] = \lambda^n$이라고 가정하면,)

$$\sum_{k=0}^{N} a_k \lambda^{n-k} = 0$$

서로다른근이라면,

$$y_h[n] = C_1\lambda_1^n + C_2\lambda_2^n + \cdots + C_N\lambda_N^n$$

중근이라면,

$$y_h[n] = C_1\lambda_1^n + C_2 n\lambda_1^n + \cdots + C_{p_1} n^{p_1-1}\lambda_1^n + C_{p_1+1}\lambda_{p_1+1}^n + C_N\lambda_N^n$$

예제) 다음 차분 방정식의 균일해를 구하라.

$$y[n]+0.5y[n-1]=0,\quad y[-1]=-1$$

풀이

균일해가 $y_h[n]=\lambda^n$이라고가정,

$$\lambda^n+0.5\lambda^{n-1}=0$$

$$\lambda^{n-1}(\lambda+0.5)=0$$

$\lambda=-0.5$ 이되고

이때 $y[-1]=-1$이므로 $C=0.5$가된다. 따라서 균일해는,

$$y_h[n]=C\lambda^n=(0.5)(-0.5)^n=(0.25)^n$$

예제) 다음 1차 차분 방정식의 특수해를 구하라.

$$y[n]+0.8y[n-1]=x[n], \quad x[n]=u[n]$$

($u[n]$은 앞에서 배운 단위계단 함수이다.)

풀이

입력이 $n \geq 0$에서 상수이므로 특수해의 형태도 상수라가정,

$$y_p[n]=Ku[n]$$

$$Ku[n]+0.5Ku[n-1]=u[n]$$

$n \geq 1$에대해

$K+0.5K=1$ 따라서, $K=\dfrac{1}{1.5}$

특수해는 $y_p[n]=\dfrac{1}{1.5}u[n]$

## - Problem

$$y(n)-0.25y(n-1)-0.125y(n-2)=0$$
$$y(-1)=1,\ y(-2)=0$$

$$1-0.25z^{-1}-0.125z^{-2}=0$$

$$(1-0.5z^{-1})(1+0.25z^{-1})=0 \rightarrow r_1=0.5,\ r_2=-0.25$$

$$y_{IC}(n)=C_1(0.5)^n+C_2(-0.25)^n$$

$$\begin{array}{l} C_1(0.5)^{-1}+C_2(-0.25)^{-1}=1 \\ C_1(0.5)^{-2}+C_2(-0.25)^{-2}=0 \end{array} \rightarrow C_1=0.333,\ C_2=-0.083$$

$$y_{IC}(n)=0.333(0.5)^n-0.083(-0.25)^n$$

## - Definition

Two sided z-transform

$$Z[x(n)] = X(z) = \sum_{n=-\infty}^{\infty} x(n) z^{-n}$$

One sided z-transform for causal

$$X(z) = \sum_{n=0}^{\infty} x(n) z^{-n}$$

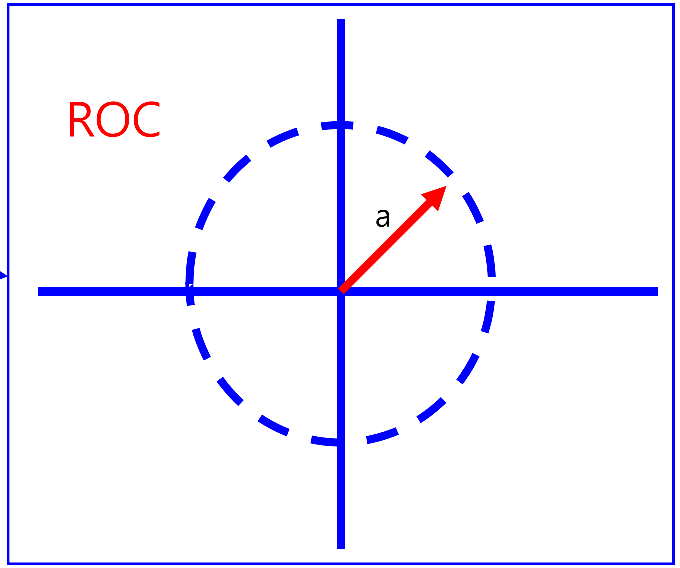
- ROC (Region Of Convergence)

$$x(n) = \begin{cases} a^n & n \geq 0 \\ 0 & n < 0 \end{cases}$$

$$X(z) = \sum_{n=-\infty}^{\infty} a^n u(n) z^{-n} = \sum_{n=0}^{\infty} (az^{-1})^n$$

$$\left|az^{-1}\right| < 1, \qquad |z| > a$$

$$X(z) = \frac{1}{1-az^{-1}} = \frac{z}{z-a}$$



[예제 7.3] 다음 두 이산 신호의 $z$−변환을 구해보자.

$$x[n]=\begin{cases}\left(\frac{1}{2}\right)^n, & n\ge 0\\ 0, & n<0\end{cases}$$

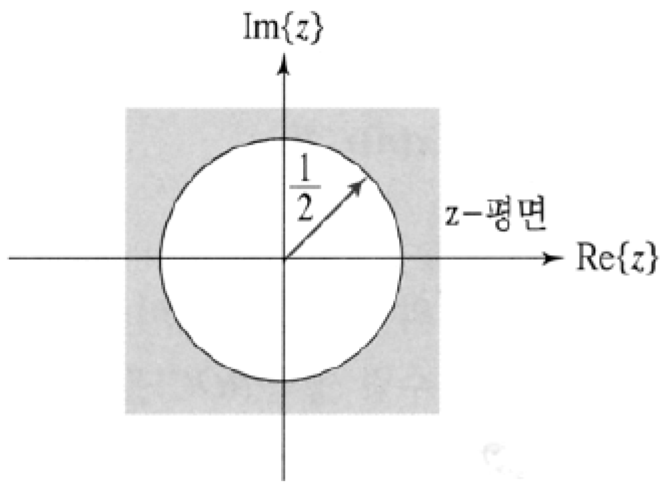
$$X(z)=1+\frac{1}{2}z^{-1}+\left(\frac{1}{2}\right)^2 z^{-2}+\left(\frac{1}{2}\right)^3 z^{-3}+\cdots+\left(\frac{1}{2}\right)^n z^{-n}+\cdots$$

$$=\sum_{n=0}^{\infty}\left(\frac{1}{2}\right)^n z^{-n}=\sum_{n=0}^{\infty}\left(\frac{1}{2}z^{-1}\right)^n$$

$$1+A+A^2+A^3+\cdots=\frac{1}{1-A} \quad \text{만약 } |A|<1$$

$$\left|\frac{1}{2}z^{-1}\right|<1$$

$$X(z)=\frac{1}{1-\frac{1}{2}z^{-1}}, \quad ROC\ :\ |z|>\frac{1}{2}$$

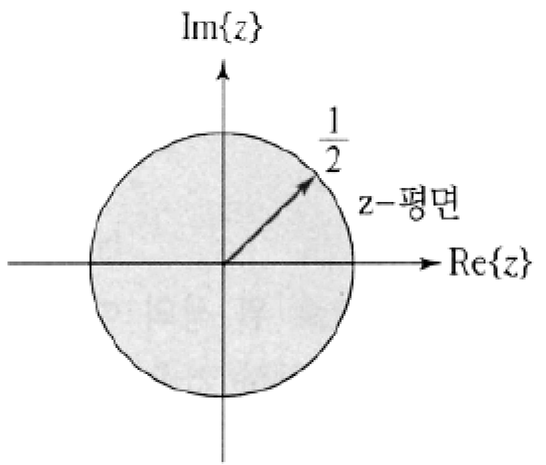


$$y[n] = \begin{cases} -\left(\frac{1}{2}\right)^n, & n < 0 \\ 0, & n \geq 0 \end{cases}$$

$$Y(z) = \sum_{n=-\infty}^{-1} -\left(\frac{1}{2}\right)^n z^{-n} = -\sum_{n=-\infty}^{-1} \left(\frac{1}{2} z^{-1}\right)^n = -\sum_{n=1}^{\infty} (2z)^n$$

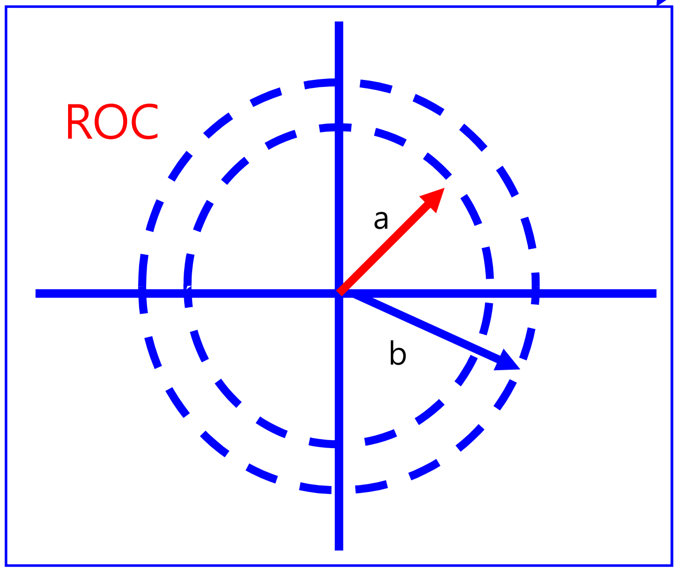
$$|2z| < 1$$

$$Y(z) = -\frac{2z}{1-2z} = \frac{1}{1-\frac{1}{2}z^{-1}}, \quad ROC : |z| < \frac{1}{2}$$



$$x(n) = \begin{cases} a^n & n \geq 0 \\ -b^n & n \leq -1 \end{cases}$$

$$X(z) = \frac{z}{z-a} + \frac{z}{z-b} = \frac{z(2z-a-b)}{(z-a)(z-b)}, \quad |a| < |z| < |b|$$




HOME WORK : Description

[예제 7.4] 다음 이산 신호의 $z$−변환을 구하여라.

$$x[n] = a^n u[n] + b^n u[-n-1]$$

$$X(z) = \sum_{n=0}^{\infty} a^n z^{-n} + \sum_{n=-\infty}^{-1} b^n z^{-n}$$

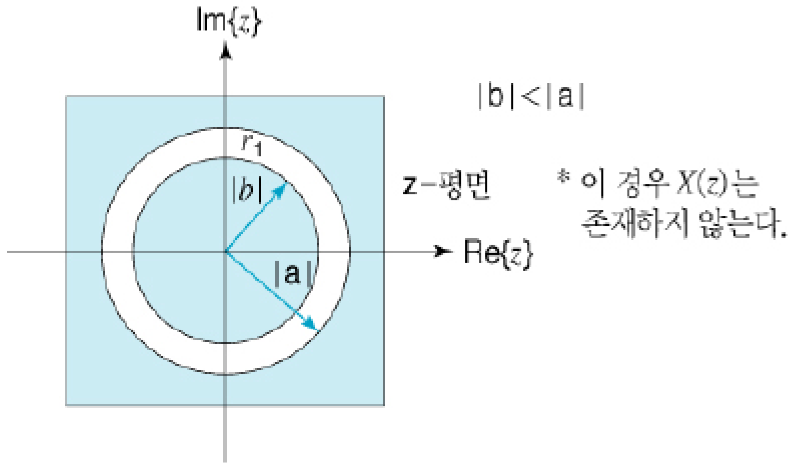
$$= \sum_{n=0}^{\infty} (az^{-1})^n + \sum_{i=1}^{\infty} (b^{-1} z)^i$$

$$|az^{-1}| < 1 \qquad |z| > |a|$$

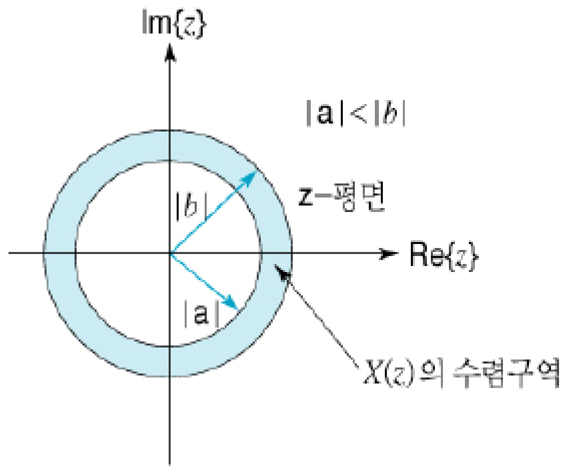
$$|b^{-1} z| < 1 \qquad |z| < |b|$$

$$X(z) = \frac{1}{1-az^{-1}} - \frac{1}{1-bz^{-1}}$$

$$= \frac{b-a}{a+b-z-abz^{-1}}, \quad ROC : |a| < |z| < |b|$$




(a)




(b)

$$x(n) = A\cos n\omega \cdot u(n)$$

$$X(z) = \sum_{n=0}^{\infty} A\cos n\omega z^{-n}$$

$$= A\sum_{n=0}^{\infty}\left[\frac{e^{j\omega n} + e^{-j\omega n}}{2}\right] z^{-n}$$

$$= \frac{A}{2}\sum_{n=0}^{\infty} e^{j\omega n} z^{-n} + \frac{A}{2}\sum_{n=0}^{\infty} e^{-j\omega n} z^{-n}$$

$$= \frac{A}{2}\sum_{n=0}^{\infty}\left(e^{j\omega} z^{-1}\right)^n + \frac{A}{2}\sum_{n=0}^{\infty}\left(e^{-j\omega} z^{-1}\right)^n$$

$$= \frac{A}{2}\left[\frac{1}{1-e^{j\omega}z^{-1}} + \frac{1}{1-e^{-j\omega}z^{-1}}\right] = \frac{Az(z-\cos\omega)}{z^2 - 2z\cos\omega + 1}$$

- Linearity

$$Z[x_1(n)] = X_1(z)$$
$$Z[x_2(n)] = X_2(z)$$
$$\Rightarrow Z[ax_1(n) + bx_2(n)] = aX_1(z) + bX_2(z)$$

- Shift Property

$$Z[x(n-m)] = z^{-m}X(z)$$

- Convolution

$$y(n) = x(n) * h(n) \quad \rightarrow \quad Y(z) = X(z)H(z)$$

- **예제22.22**: $e^{-k}+k$의 z 변환을 구하라.
  - ✓ 풀이: $Z\left\{e^{-k}+k\right\}=Z\left\{e^{-k}\right\}+Z\{k\}=\frac{z}{z-e^{-1}}+\frac{z}{(z-1)^2}$
- **예제22.24**: t=kT(k=정수)에서 표본화된 함수 $f(t)=2t^2$의 z 변환을 구하라.
  - ✓ 풀이:

$$f[k]=2(kT)^2=2T^2k^2$$

$$Z\{f[k]\}=Z\left\{2T^2k^2\right\}=2T^2Z\left\{k^2\right\}$$

$$=2T^2\frac{z(z+1)}{(z-1)^2}$$

- 제1 이동 이론
  - ✓ 수열 f[k]의 z-변환이 F(z)이면, (i=양의 정수)

$$Z\{f[k+i]\} = z^i F(z) - \left(z^i f[0] + z^{i-1} f[1] + \cdots + zf[i-1]\right)$$

  - ✓ i=2일 때,

$$Z\{f[k+2]\} = z^2 F(z) - z^2 f[0] - zf[1]$$

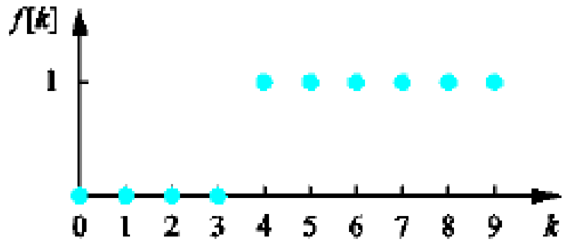
  - ✓ i=1일 때,

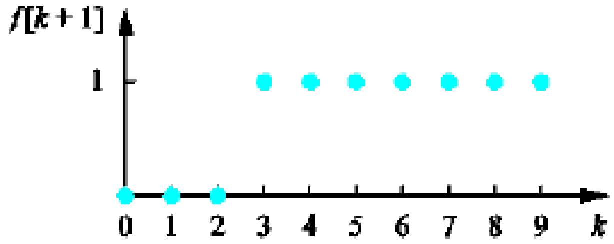
$$Z\{f[k+1]\} = zF(z) - zf[0]$$

- 증명: $Z\{f[k+1]\} = \sum_{k=0}^{\infty} f[k+1] z^{-k} = z \sum_{k=0}^{\infty} f[k+1] z^{-(k+1)}, \; let\ m = k+1$

$$= z \sum_{m=1}^{\infty} f[m] z^{-m} = z\left\{ \sum_{m=0}^{\infty} f[m] z^{-m} - f[0] \right\} = zF(z) - zf[0]$$

- **예제22.25**: 주어진 f[k]에 대해서 f[k+1]을 구하고, 다음 식을 증명하시오. $Z\{f[k+1]\} = zF(z) - zf[0]$
  - ✓ 풀이:




$$Z\{f[k]\} = \sum_{k=0}^{\infty} f[k]z^{-k} = \sum_{k=4}^{\infty} z^{-k} = z^{-4}\left\{1 + \frac{1}{z} + \frac{1}{z^2} + \cdots\right\}$$

$$= z^{-4}\frac{1}{1 - 1/z} = z^{-4}\frac{z}{z-1} = \frac{1}{z^3(z-1)}$$

$$Z\{f[k+1]\} = \sum_{k=3}^{\infty} z^{-k} = z^{-3}\left\{1 + \frac{1}{z} + \frac{1}{z^2} + \cdots\right\} = \frac{1}{z^2(z-1)}$$

$$\therefore Z\{f[k+1]\} = zZ\{f[k]\} - f[0],\ where\ f[0] = 0$$

- 제2 이동 이론
  - ✓ f(t)u(t)를 T간격으로 표본화하여 오른쪽으로 i번의 표본간격 이동을 하면, f(t-iT)u(t-iT)로 표현된다. 이를 수열로 나타내면,

$$f[k-i]u[k-i],\ k \in N$$

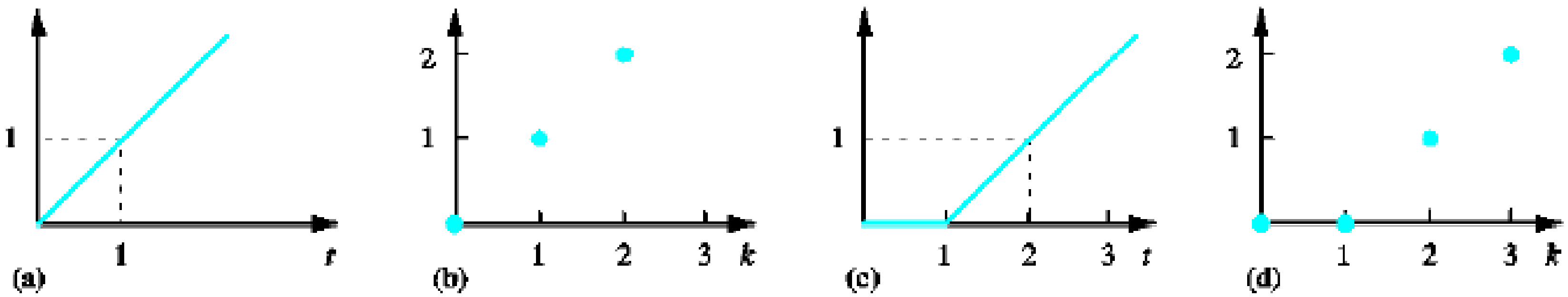
  - ✓ 이때, $Z\{f[k-i]u[k-i]\} = z^{-i}F(z), \quad i \in N^{+}$
    - 증명: $Z\{f[k-i]u[k-i]\} = \sum_{k=0}^{\infty} f[k-i]u[k-i]z^{-k}, let\ m = k-i$

$$= \sum_{m=-i}^{\infty} f[m]u[m]z^{-(m+i)} = \sum_{m=0}^{\infty} f[m]z^{-(m+i)}$$

$$= z^{-i}\sum_{m=0}^{\infty} f[m]z^{-m}$$

$$= z^{-i}F(z)$$

- **예제22.26**: 함수 t·u(t)는 k·u[k]를 얻기 위해 T=1 간격으로 표본화되었다. 이 표본은 하나의 표본화 간격으로 오른쪽으로 이동하여, (k-1)u[k-1]을 얻는다. 이 수열의 z-변환을 구하라.




✓ 풀이:

$$Z\{k\} = \frac{z}{(z-1)^2}$$

$$Z\{(k-1)u[k-i]\} = z^{-1}\frac{z}{(z-1)^2} = \frac{1}{(z-1)^2}$$

- **예제22.28**: z-변환이 1/(z-1)인 수열을 구하라.
  - ✓ 풀이:

$$\frac{1}{z-1} = z^{-1}\frac{z}{z-1}$$

$$= z^{-1}Z\{u[k]\} = Z\{u[k-1]\}$$

- **예제22.29**: z-변환이 아래와 같은 수열을 구하라.

$$\frac{1}{z^2(z-1)^2}$$

  - ✓ 풀이:

$$\frac{1}{z^2(z-1)^2} = \frac{1}{z^3}\frac{z}{(z-1)^2} = \frac{1}{z^3}Z\{k\}$$

$$= z^{-3}Z\{k\}$$

$$= Z\{(k-3)u[k-3]\}$$

$$\delta[k] \Rightarrow 1 \qquad u[k] \Rightarrow \frac{z}{z-1} \qquad k \Rightarrow \frac{z}{(z-1)^2}$$

$$e^{-ak} \Rightarrow \frac{z}{z-e^{-a}} \qquad a^k \Rightarrow \frac{z}{z-a} \qquad ka^k \Rightarrow \frac{az}{(z-a)^2} \qquad k^2a^k \Rightarrow \frac{az(z+a)}{(z-a)^3}$$

$$k^2 \Rightarrow \frac{z(z+1)}{(z-1)^3} \qquad k^3 \Rightarrow \frac{z(z^2+4z+1)}{(z-1)^4}$$

$$\sin ak \Rightarrow \frac{z\sin a}{z^2-2z\cos a+1} \qquad \cos ak \Rightarrow \frac{z(z-\cos a)}{z^2-2z\cos a+1}$$

$$e^{-ak}\sin bk \Rightarrow \frac{ze^{-a}\sin b}{z^2-2ze^{-a}\cos b+e^{-2a}} \qquad e^{-ak}\cos bk \Rightarrow \frac{z^2-ze^{-a}\cos b}{z^2-2ze^{-a}\cos b+e^{-2a}}$$

## - Inverse z-Transform

$$x(n) = Z^{-1}[X(z)] = \frac{1}{2\pi j}\oint_{\Gamma} X(z) z^{n-1} dz$$

$$x(n) = \begin{cases} a^n & n \geq 0 \\ 0 & n < 0 \end{cases}$$

$$X(z) = \frac{1}{1 - az^{-1}} = \frac{z}{z-a}$$

Very! Very! Difficult….

So…. We use RESIDUE THEOREM

$$X(z) = \frac{0.5z}{(z-1)(z-0.5)}$$

$$\frac{X(z)}{z} = \frac{k_1}{z-1} + \frac{k_2}{z-0.5}$$

$$k_1 = \frac{X(z)}{z}(z-1)\Big|_{z=1} = 1$$

$$k_2 = \frac{X(z)}{z}(z-0.5)\Big|_{z=0.5} = -1$$

$$X(z) = \frac{z}{z-1} - \frac{z}{z-0.5}$$

$$x(n) = 1 - 0.5^n$$

$$Y(z) = \frac{6z^2 - 10z + 2}{z^2 - 3z - 2}$$

$$\frac{Y(z)}{z} = \frac{6z^2 - 10z + 2}{z(z-1)(z-2)} = \frac{C_0}{z} + \frac{C_1}{z-1} + \frac{C_2}{z-2}$$

$$Y(z) = 1 + \frac{2z}{z-1} + \frac{3z}{z-2}$$

$$|z| > 2$$

$$y(n) = \delta(n) + [2(1)^n + 3(2)^n]u(n)$$

## [예제7.10] 다음 z-변환의 역변환을 부분 분수 전개에 의한 방법으로 구하여라.

$$X(z) = \frac{1}{1 - 1.5z^{-1} + 0.5z^{-2}}, \quad ROC : |z| > 1$$

$$X(z) = \frac{z^2}{z^2 - 1.5z + 0.5}$$

$$X(z) = \frac{z^2}{(z-1)(z-0.5)} = \frac{A_1 z}{z-1} + \frac{A_2 z}{z-0.5}$$

$$z^2 = (z-0.5)A_1 z + (z-1)A_2 z$$

$$1 = (1-0.5)A_1 \longleftarrow z = 1$$

$$0.5^2 = (0.5-1)A_2(0.5) \longleftarrow z = 0.5$$

$$X(z) = \frac{2z}{z-1} - \frac{z}{z-0.5} \xrightarrow{\text{수렴 영역 } |z|>1} x[n] = 2u[n] - (0.5)^n u[n]$$

$$u[n] \rightarrow \frac{z}{z-1}$$

$$a^n \rightarrow \frac{z}{z-a}$$

➢ 예)

$$X(z) = \frac{1 + 0.2z^{-1}}{1 - 1.7z^{-1} + 0.6z^{-2}}.$$

$$X(z) = \frac{1 + 0.2z^{-1}}{(1 - 0.5z^{-1})(1 - 1.2z^{-1})} = \frac{r_1}{1 - 0.5z^{-1}} + \frac{r_2}{1 - 1.2z^{-1}}$$

$$r_1 = X(z)(1 - 0.5z^{-1})\Big|_{z=0.5} = \frac{1 + 0.2z^{-1}}{1 - 1.2z^{-1}}\Bigg|_{z=0.5} = -1,$$

$$r_2 = X(z)(1 - 1.2z^{-1})\Big|_{z=1.2} = \frac{1 + 0.2z^{-1}}{1 - 0.5z^{-1}}\Bigg|_{z=1.2} = 2.$$

$$X(z) = -\frac{1}{1 - 0.5z^{-1}} + \frac{2}{1 - 1.2z^{-1}}$$

# 분자가 분모에 비해 차수가 같거나 높을 경우

- 차수를 낮도록 만든 다음 부분 분수 전개

$$X(z) = \frac{\sum_{k=0}^{M} b_k z^{-k}}{\sum_{k=0}^{N} a_k z^{-k}} = G(z) + \frac{N'(z)}{D(z)}, \quad |z| > r_{\max}$$

$$X(z) = \frac{2 - 3.5z^{-1} + 2.5z^{-2} - 0.5z^{-3}}{1 - 1.5z^{-1} + 0.5z^{-2}}, \quad |z| > 1.$$

$$\begin{array}{r}
\overbrace{-z^{-1} + 2}^{G(z)} \\
\underbrace{0.5z^{-2} - 1.5z^{-1} + 1}_{D(z)} \overline{\big)\, -0.5z^{-3} + 2.5z^{-2} - 3.5z^{-1} + 2} \\
\underline{-0.5z^{-3} + 1.5z^{-2} - \quad z^{-1}} \\
z^{-2} - 2.5z^{-1} + 2 \\
\underline{z^{-2} - \quad 3z^{-1} + 2} \\
\underbrace{0.5z^{-1}}_{N'(z)}.
\end{array}$$

$$\therefore\ G(z) = -z^{-1} + 2$$

$$\frac{N'(z)}{D(z)} = \frac{0.5z^{-1}}{1 - 1.5z^{-1} + 0.5z^{-2}} = \frac{1}{1 - z^{-1}} - \frac{1}{1 - 0.5z^{-1}},$$

$$\therefore\ X(z) = -z^{-1} + 2 + \frac{1}{1 - z^{-1}} - \frac{1}{1 - 0.5z^{-1}}, \quad |z| > 1$$

$$x[n] = 2\delta[n] - \delta[n-1] + \{1 - 0.5^n\} u[n]$$

- **예제22.31**: 다음 z-변환에 대해서 f[k]를 구하라.

$$F(z) = \frac{z+3}{z-2}$$

✓ 풀이:

$$F(z) = \frac{z+3}{z-2} = \frac{z}{z-2} + \frac{3}{z-2}$$

$$= \frac{z}{z-2} + 3z^{-1}\frac{z}{z-2} = Z\{2^k\} + 3z^{-1}Z\{2^k\}$$

$$= Z\{2^k\} + 3Z\{2^{k-1}u[k-1]\}$$

$$f[k] = 2^k + 3\cdot 2^{k-1}u[k-1] = 2\cdot 2^{k-1} + 3\cdot 2^{k-1}u[k-1]$$

$$= 2\cdot 2^{k-1}u[k] + 3\cdot 2^{k-1}u[k-1]$$

$$= 2\cdot 2^{k-1}\{\delta[k] + u[k-1]\} + 3\cdot 2^{k-1}u[k-1]$$

$$= 2\cdot 2^{k-1}\delta[k] + \{2\cdot 2^{k-1} + 3\cdot 2^{k-1}\}u[k-1]$$

$$= 1 + 5\cdot 2^{k-1}u[k-1]$$

- **예제22.32**: 다음 z-변환에 대해서 f[k]를 구하라.

$$F(z)=\frac{2z^2-z}{(z-5)(z+4)}$$

✓ 풀이: 부분분수로 정리

$$F(z)=\frac{2z^2-z}{(z-5)(z+4)}=\frac{z(2z-1)}{(z-5)(z+4)}$$

$$=z\left\{\frac{1}{z-5}+\frac{1}{z+4}\right\}=\frac{z}{z-5}+\frac{z}{z+4}$$

$$\therefore f[k]=5^k+(-4)^k$$

- **예제22.34**: 이항정리를 이용하여 다음 z-변환에 대해서 f[k]를 구하라.

$$F(z)=\frac{z^3}{(z-1)^3}$$

✓ 풀이:

$$F(z)=\frac{z^3}{(z-1)^3}=\left(\frac{z}{z-1}\right)^3=\left(\frac{z-1}{z}\right)^{-3}=\left(1-\frac{1}{z}\right)^{-3},\quad |z^{-1}|<1$$

$$=1+(-3)\left(-\frac{1}{z}\right)+\frac{(-3)(-4)}{2!}\left(-\frac{1}{z}\right)^2+\frac{(-3)(-4)(-5)}{3!}\left(-\frac{1}{z}\right)^3+\cdots$$

$$=1+\frac{3}{z}+\frac{6}{z^2}+\frac{10}{z^3}+\frac{15}{z^4}+\cdots$$

$$\therefore f[k]=\frac{(k+2)(k+1)}{2},\quad k\ge 0$$

## - The Solution of linear Diiference Equations

$$x(n) - 1.01x(n-1) = 50, x(-1) = 0, n \geq 0$$

$$X(z) - 1.01[z^{-1}X(z) + x(-1)] = \frac{50}{1 - z^{-1}}$$

$$X(z) = \frac{1}{1 - 1.01z^{-1}} \cdot \frac{50}{1 - z^{-1}} = \frac{z}{z - 1.01} \cdot \frac{50z}{z - 1}$$

$$\frac{X(z)}{z} = \frac{1}{z - 1.01} \cdot \frac{50z}{z - 1} = \frac{101}{z - 1.01} - \frac{100z}{z - 1}$$

$$X(z) = 50\left[\frac{101z}{z - 1.01} - \frac{100z}{z - 1}\right]$$

$$x(n) = 50[101(1.01)^n - 100]u(n)$$

- **예제22.35**: 차분방정식 y[k+1]-3y[k]=0, y[0]=4를 구하시오.
  - ✓ 풀이: 방정식 양단에 z-변환을 하면,

$$Z\{y[k+1]\}-3Z\{y[k]\}=zY(z)-zy[0]-3Y(z)$$
$$=zY(z)-4z-3Y(z)=0$$
$$(z-3)Y(z)=4z$$
$$Y(z)=\frac{4z}{z-3}$$
$$\therefore y[k]=4(3)^k$$

- **예제22.36**: 다음 차분 방정식을 풀어라.

$$y[k+2]-5y[k+1]+6y[k]=0, \quad y[0]=0, y[1]=2$$

✓ 풀이: 양단에 z-변환을 수행한 후, 역변환을 수행한다.

$$Z\{y[k+2]-5y[k+1]+6y[k]\}=0, \quad y[0]=0, y[1]=2$$

$$z^2Y(z)-z^2y[0]-zy[1]-5\{zY(z)-zy[0]\}+6Y(z)=0$$

$$(z^2-5z+6)Y(z)=2z$$

$$Y(z)=\frac{2z}{z^2-5z+6}$$

$$=\frac{2z}{(z-2)(z-3)}=\frac{2z}{z-3}-\frac{2z}{z-2}$$

$$\therefore y[k]=2(3)^k-2(2)^k$$

## - The Relationship with Laplace Transform

$$x_s = \sum_{n=0}^{\infty} x(nT)\delta(t - nT)$$

$$X_s(s) = x(0) + x(T)e^{-sT} + x(2T)e^{-2sT} + \cdots$$

$$= \sum_{n=0}^{\infty} x(nT)e^{-nsT}$$

$$e^{sT} = z$$